ArtiCheck: well-typed generic fuzzing for module interfaces

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Abstract

In spite of recent advances in program certification, testing remains a widely-used component of the software development cycle. Various flavors of testing exist: popular ones include *unit testing*, which consists in manually crafting test cases for specific parts of the code base, as well as *quickcheck-style* testing, where instances of a type are automatically generated to serve as test inputs.

These classical methods of testing can be thought of as *internal* testing: the test routines access the internal representation of the data structures to be checked. We propose a new method of *external* testing where the library only deals with a *module interface*. The data structures are exported as *abstract types*; the testing framework behaves like regular client code and combines functions exported by the module to build new elements of the various abstract types. Any counter-examples are then presented to the user.

Categories and Subject Descriptors D.2.5 [*Software Engineering*]: Testing and Debugging

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1. Introduction

Software development is hard. Industry practices still rely, for the better part, on tests to ensure the functional correctness of programs. Even in more sophisticated circles, such as the programming language research community, not everyone has switched to writing all their programs in Coq. Testing is thus a cornerstone of the development cycle. Moreover, even if the end goal is to fully certify a program using a proof assistant, it is still worthwhile to eliminate bugs early by running a cheap, efficient test framework.

Testing boils down to two different processes: generating test cases for test suites; then verifying that user-written assertions and specifications of program parts are not falsified by the test suites.

QuickCheck [5] is a popular, efficient tool for that purpose. First, it provides a combinator library based on type-classes to build test case generators. Second, it provides a principled way for users to specify properties over functions. For instance, users may write predicates such as “reverse is an involution”. Then, the QuickCheck framework is able to create *instances* of the type being tested, e.g. lists of integers. The predicate is tested against these test cases, and any counter-example is reported to the user.

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Our novel approach is motivated by some limitations of the QuickCheck framework. First, data-structures often have *internal invariants* that would not be broken when using the API of the data-structure. Thus, testing one particular function of such an API requires generating test-cases that meet these invariants. Yet, writing *good* generators for (involved) data-structures is tedious, or even plain hard. Consider the simple problem of generating binary search trees (BST), for instance. Being smarter than merely generating trees and filtering out those which are not search trees requires reimplementing a series of insertions and deletions into BSTs. But these functions are certainly already part of the code that is tested!

We argue that this low-level manipulation could be taken care of by the testing framework. That is, we argue that the framework should be able, by itself, to combine functions exported by the module we wish to test in order to build instances of the data-types defined in the same module. If the module exports the properties and the invariants that should hold, then the testing framework needs not see the implementation. In a nutshell, we want to move towards an *external testing* of abstract modules.

In the present document, we describe a library that does precisely that, dubbed ArtiCheck. The library is written in OCaml. While QuickCheck uses type-classes as a host language feature to conduct value generation, we show how to implement the proof search in library code – while remaining both type-safe and generic over the tested interfaces, thanks to GADTs.

2. The essence of external testing

In the present section, we illustrate the essential idea of external testing through a simple example, which is that of a module `SIList` whose type `t` represents sorted integer lists. The invariant is maintained by making `t` abstract and requiring the user to go through the exported functions `empty` and `add`.

This section, unfolding from the initial example, introduces the key ideas of external testing: a GADT type that describes well-typed applications in the simply-typed lambda calculus; a description of module signatures that we wish to test; type descriptors that record all the instances of a type that we managed to construct.

The point of view adopted in this section is intendedly simplistic. The design, as presented here, contains several obvious shortcomings. It allows, nonetheless, for a high-level overview of our principles, and paves the way for a more thorough discussion of our design (§3).

Here is the signature for our module of sorted integer lists. The `sorted` function represents the *invariant* that the module intends to preserve. The module admits a straightforward implementation, which we also show.

```
module SIList: sig
  type t
  val empty: t
  val add: t -> int -> t
  val sorted: t -> bool
```

```
end = struct
  type t = int list
  let empty = []
  let rec add x = function
    | [] -> [x]
    | t::q -> if t<x then t::add x q else x::t::q
  let rec sorted = function
    | [] | [_] -> true
    | t1::(t2::_ as q) -> t1 <= t2 && sorted q
end
```

Roughly speaking, our goal is to generate, as if we were *client code* of the module, instances of type `t` using only the functions exported by the module. Therefore, one of our first requirements is a data structure for keeping track of the `t`'s created so far. We also need to keep track of the integers we have generated so far, since they are necessary to call the `add` function: ArtiCheck will thus manipulate several `ty`'s for all the types it handles.

```
type 'a ty = { (* Other implementation details omitted *)
  mutable enum: 'a list;
  fresh: ('a list -> 'a) option; }
```

A type descriptor `'a ty` keeps track of all the *instances* of `'a` we have created so far in its `enum` field. Built-in types such as `int` do not belong to the set of types whose invariants we wish to check. For such types, we provide a `fresh` function that generates an instance different from all that we have generated so far.

From the point of view of the client code, all we can do is combine `add` and `empty` to generate new instances. ArtiCheck, as a fake client, should thus behave similarly and automatically perform repeated applications of `add` so as to generate new instances. We thus need a description of what combinations of functions are legal for ArtiCheck to perform.

In essence, we want to represent well-typed applications in the simply-typed lambda-calculus. This can be embedded in OCaml using generalized algebraic data types (GADTs). We define the GADT `('a, 'b) fn`, which describes ways to generate instances of type `'b` using a function of type `'a`. We call it a *function descriptor*.

```
type (_,_) fn =
| Ret: 'a ty -> ('a,'a) fn
| Fun: 'a ty * ('b, 'c) fn -> ('a -> 'b, 'c) fn
(* Helpers for creating [fn]'s. *)
let (@->) ty fd = Fun (ty,fd)
let returning ty = Ret ty
```

The `Ret` case describes a constant value, which has type `'a` and produce one instance of type `'a`. For reasons that will soon become apparent, we also record the descriptor of type `'a`. `Fun` describes the case of a function from `'a` to `'b`: using the descriptor of type `'a`, we can apply the function to obtain instances of type `'b`; combining that with the other `('b, 'c) fn` gives us a way to produce elements of type `'c`, hence the `('a -> 'b, 'c) fn` conclusion.

```
let rec eval : type a b. (a,b) fn -> a -> b list =
  fun fd f ->
    match fd with
    | Ret _ -> [f]
    | Fun (ty,fd) -> List.flatten (
        List.map (fun e -> eval fd (f e)) ty.enum)
let rec codom : type a b. (a,b) fn -> b ty =
  function
    | Ret ty -> ty
    | Fun (_,fd) -> codom fd
```

The `eval` function is central: taking a function descriptor `fd`, it recurses over it, thus refining the type of its argument `f`. The use of GADTs allows us to statically prove that the `eval` function only ever produces instances of type `b`. The `codom` function allows one to find the type descriptor associated to the return value (the codomain) of an `fn`.

Using the two functions above, it then becomes trivial to generate new instances of `'b`.

```
let use (fd: ('a, 'b) fn) (f: 'a): unit =
  let prod, ty = eval fd f, codom fd in
  List.iter (fun x ->
    if not (List.mem x ty.enum)
    then ty.enum <- x::ty.enum
  ) prod
```

The function takes a function descriptor along with a matching function. The `prod` variable contains all the instances of `'b` we just managed to create; `ty` is the descriptor of `'b`. We store the new instances of `'b` in the corresponding type descriptor.

In order to wrap this up nicely, one can define *signature descriptors*. An entry in a signature descriptor is merely a function of a certain type `'a` along with its corresponding function descriptor. Once this is done, the user can finally call our library to generate random instances and test the functions found in the signature description.

```
type sig_elem = Elem : ('a,'b) fn * 'a -> sig_elem
type sig_descr = (string * sig_elem) list
let si_t : SIList.t ty  = ...
let int_t = ... (* integers use a [fresh] function*)
let sig_of_silist = [
  ("empty", Elem (returning si_t, SIList.empty));
  ("add", Elem (int_t @-> si_t @-> returning si_t, SIList.add)); ]
let _ =
  Arti.generate sig_of_silist;
  assert (Arti.counter_example si_t SIList.sorted = None)
```

3. Implementing ArtiCheck

The simplistic design we introduced in §2 conveys the main ideas behind ArtiCheck, yet fails to address a wide variety of problems. The present section reviews the issues with the current design and incrementally addresses them.

3.1 A better algebra of types

Type descriptions only model function types so far. We want to support products and sums, to be able to generate them when they appear in function arguments, and to use them to discover new values when they are returned as function results.

One of the authors naïvely suggested that the data type be extended with cases for products and sums, such as:

```
| Prod: ('a,'c) fn * ('b,'c) fn -> ('a * 'b,'c) fn
```

It turns out that the constructor above does not model product arguments. If `'a` is `int -> int` and `'b` is `int -> float`, not only do the `'c` parameters fail to match, but the `'a * 'b` parameter in the conclusion represents a pair of functions, rather than a function that returns a pair! Another snag is that the type of `eval` makes no sense in the case of a product. If the first parameter of type `('a, 'b) fn` represents a way to obtain a `'b` from the product type `'a`, then what use is the second parameter of `eval`?

Datatypes and function types are used in fundamentally different ways by the generator, which suggests splitting the `fn` type into two distinct GADTs – inspired by the focusing literature [9]. The GADT `('a, 'b) negative` (`neg` for short) represents a *computation* of type `'a` that produces a result of type `'b`. The GADT `'a positive` (`pos` for short) represents a *value*, that is, the result of a computation.

```
type (_, _) neg =
| Fun : 'a pos * ('b, 'c) neg -> ('a -> 'b, 'c) neg
| Ret : 'a pos -> ('a, 'a) neg
and _ pos =
```

```
| Ty : 'a ty -> 'a pos
| Sum : 'a pos * 'b pos -> ('a, 'b) sum pos
| Prod : 'a pos  * 'b pos -> ('a * 'b) pos
| Bij : 'a pos * ('a, 'b) bijection -> 'b pos
and ('a, 'b) sum = L of 'a | R of 'b
and ('a, 'b) bijection = ('a -> 'b) * ('b -> 'a)
```

The `pos` type represents first-order data types: products, sums and atomic types, that is, whatever is on the rightmost side of an arrow. We provide an injection from positive to negative types via the `Ret` constructor: a value of type `'a` is also a constant computation.

We do *not* provide an injection from negative types to positive types, which would be necessary to model nested arrows, that is, higher-order functions. One can take the example of the `map` function, which has type `('a -> 'b) -> 'a list -> 'b list`: we explicitly disallow representing the `'a -> 'b` part as a `Fun` constructor, as it would require us to synthesize instances of a function type (see §6 for a discussion). Note that the user can still use the `Ty` constructor to represent `'a -> 'b` as an atomic type, initialized with its own test functions.

Our GADT does not accurately model tagged, n-ary sums of OCaml, nor records with named fields. We thus add a last `Bij` case; it allows the user to provide a two-way mapping between a built-in type (say, `'a option`) and its ArtiCheck representation (`() + 'a`). That way, ArtiCheck can work with regular OCaml data types by converting them back-and-forth.

This change of representation incurs some changes on our evaluation functions as well. The `eval` function is split into several parts, which we detail right below.

```
let rec apply: type a b. (a, b) neg -> a -> b list =
  fun ty v -> match ty with
  | Fun (p, n) ->
      produce p |> concat_map (fun a -> apply n (v a))
  ...
and produce: type a. a pos -> a list =
  fun ty -> match ty with
  | Ty ty -> ty.enum
  | Prod (pa, pb) ->
      cartesian_product (produce pa) (produce pb)
  ...
let rec destruct: type a. a pos -> a -> unit =
  function
  | Ty ty -> (fun v -> remember v ty)
  | Prod (ta, tb) -> (fun (a, b) ->
      destruct ta a; destruct tb b)
  ...
(* Putting it all together *)
let _ = ...
  let li = apply fd f in
  List.iter destruct li; ...
```

Let us first turn to the case of *values*. In order to understand what ArtiCheck ought to do, one may ask themselves what the user can do with values. The user may destruct them: given a pair of type `'a * 'b`, the user may keep the first element only, thus obtaining a new `'a`. The same goes for sums. We thus provide a `destruct` function, which breaks down positives types by pattern-matching, populating the descriptions of the various types it encounters as it goes. (The `remember` function records all instances we haven't encountered yet in the type descriptor `ty`.)

Keeping this in mind, we must realize that if a function takes an `'a`, the user may use any `'a` it can produce to call the function. For instance, in the case that `'a` is a product type `'a1 * 'a2`, then *any* pair of `'a1` and `'a2` may work. We introduce a function called `produce`, which reflects the fact the user may choose any possible pair: the function exhibits the entire set of instances we can build for a given type.

Finally, the `apply` function, just like before, takes a *computation* along with a matching description, and generates a set of `b`. However, it now relies on `product` to exhaustively exhibit all possible arguments one can pass to the function.

We are now able to accurately model a calculus rich enough to test realistic signatures involving records, option types, and various ways to create functions.

3.2 Efficient construction of a set of instances

The (assuredly naïve) scenario above reveals several pain points with the current design.

- We represent our sets using lists. We could use a more efficient data structure.
- If some function takes, say, a tuple, the code as it stands will construct the set of all possible tuples, `map` the function over the set, then finally call `destruct` on each resulting element to collect instances. Naturally, memory explosion ensues. We propose a symbolic algebra for *sets of instances* that *mirrors* the structure of positive types and avoids the need for holding all possible combinations in memory at the same time.
- A seemingly trivial optimization sends us off the rails by generating an insane number of instances. We explain how to optimize further the code while still retaining a well-behaved generation.

Sets of instances The first, natural optimization that comes to mind consists in dropping lists in favor of a more sophisticated data type. For reasons that will become obvious in the following, we chose to replace lists with arbitrary containers that have the following (object-like) type:

```
type 'a bag = {
  insert : 'a -> 'a bag;
  fold : 'b . ('a -> 'b -> 'b) -> 'b -> 'b;
  cardinal : unit -> int; }
```

For instance, we use an implementation of polymorphic, persistent sets (implemented as red-black trees), as a replacement for lists.

Not holding sets in memory A big source of inefficiency is the call to the `cartesian_product` function above (§3.1). We hold in memory at the same time all possible products, then pipe them into the function calls so as to generate an even bigger set of elements. Only when the set of all elements has been constructed do we actually run `destruct`, only to extract the instances that we have created in the process.

Holding in memory the set of all possible products is too expensive. We adopt instead a *symbolic representation of sets*, where unions and products are explicitly represented using constructors. This mirrors our algebra of positive types.

```
type _ set =
  | Set     : 'a bag -> 'a set
  | Bij     : 'a set * ('a, 'b) bijection -> 'b set
  | Union   : 'a set * 'b set -> ('a,'b) sum set
  | Product : 'a set * 'b set -> ('a * 'b) set
```

This does not suppress the combinatorial explosion. The instance space is still exponentially large; what we gained by changing our representation is that we no longer hold all the "intermediary" instances in memory *simultaneously*. This allows us to write an `iter` function that constructs the various instances on-the-fly.

```
let rec iter: type a. (a -> unit) -> a set -> unit =
fun f s -> match s with
  | Set ps -> ps.fold (fun x () -> f x) ()
  | Union (pa,pb) ->
      iter (fun a -> f (L a)) pa;
      iter (fun b -> f (R b)) pb;
  | Product (pa,pb) ->
      iter (fun a -> iter (fun b -> f (a,b)) pb) pa
  | Bij (p, (proj, _)) -> iter (fun x -> f (proj x)) p
```

Sampling sets The above optimizations make it possible to build in a relatively efficient manner sets of instances that can be constructed using a small amount of function calls (let's call them *small instances*). That is, we naturally implement a breadth-first search of the instance space, which is unlikely to produce many interesting test-cases before we reach a size limit. Indeed, the distribution of instances is skewed: there are more instances obtained after `n` calls than there are after `n+1` calls. It may thus be the case that by the time we reach three of four consecutive function calls, we've hit the maximum limit of instances allowed for the type, since it often is the case that the number of instances grows exponentially.

To solve this issue, we turn to *reservoir sampling*, which is a method to choose a sample of elements from a set that is typically too big to be held in memory. The solution from the literature is a variant of Knuth's shuffle, and is quite elegant. The idea is as follows: build a collection of k elements; then for each element e of index i such that $k < i$, pick a number j at random between 1 and i: if j is less than k, replace the j-th element of the collection by e, and do nothing otherwise. In the end, each element that was added has the same probability of being in the collection.

Unfortunately, iterating over the collection to produce new instances of types biases the generation towards small instances. To understand why, imagine that the collection contains initially many 1 and that we produce new elements for the sampling process by taking the sum of two elements of the collection. The final distribution is very likely to be skewed toward small integers...

Sampling sets, done right What we are looking for is a set-like data-structure, that can be used to sample the elements that are added to it. This can be implemented quite simply using a hash-set, with fixed size buckets. The idea is that when a bucket becomes full, we drop one element. That way, we manage to keep a good balance between the size of our instances sets, and the diversity of the instances.

We have experimented with the three container structures described above: "regular" sets, "sampled" sets and hash-sets. Out of the three, the latest is the one that gives the most interesting results empirically. However, it is likely that other kind of containers, or other tunings of the exploration procedures could make "interesting" instances pop up early.

3.3 Instance generation as a fixed point computation

The `apply/destruct` combination only demonstrates how to generate new instances from one specific element of the signature. We need to iterate this on the whole signature, by feeding the new instances that we obtain to other functions that can consume them.

This part of the problem naturally presents itself as a fixpoint computation, defined by a system of equations. Equations between variables (type descriptors) describe ways of obtaining new instances (by applying functions to other type descriptors). Of course, to ensure termination, we need to put a bound on the number of generated instances. When presenting an algorithm as a fixpoint problem, it is indeed a fairly standard technique to make the lattice space artificially finite in order to obtain the termination property.

Implementing an efficient fixpoint computation is a *surprisingly interesting* activity, and we are happy to use an off-the-shelf fixpoint library, F. Pottier's `Fix` [11], to perform the work for us. `Fix` can be summarized by the signature below, obtained from user-defined instantiations of the types `variable` and `property`.

```
module Fix = sig
  type valuation = variable -> property
  type rhs = valuation -> property
  type equations = variable -> rhs
  val lfp: equations -> valuation end
```

A system of equations maps a variable to a right-hand side. Each right-hand side can be evaluated by providing a valuation so as to obtain a property. Valuations map variables to properties. Solving a system of equations amounts to calling the `lfp` function which, given a set of equations, returns the corresponding valuation.

A perhaps tempting way to fit in this setting would be to define variables to be our `'a ty` (type descriptor) and properties to be `'a lists` (the instances we have built so far); the equations derived from any signature would then describe ways of obtaining new instances by applying any function of the signature. This doesn't work as is: since there will be multiple values of `'a` (we generate instances of different types simultaneously), type mismatches are to be expected. One could, after all, use yet another GADT and hide the `'a` type parameter behind an existential variable.

```
type variable = Atom: 'a ty -> variable
type property = Props: 'a set -> property
```

The problem is that there is no way to statically prove that having an `'a var` named `x`, calling `valuation x` yields an `'a property` with a matching type parameter. This is precisely where the mutable state in the `'a ty` type comes handy: even though it is only used as the *input* parameter for the system of equations, we "cheat" and use its mutable `enum` field to store the output. That way, the `property` type needs not mention the type variable `'a` anymore, thus removing any typing difficulty – or the need to change the interface of `Fix`.

We still, however, need the `property` type to be a rich enough lattice to let `Fix` decide when to stop iterating: it should come with equality- and maximality-checking functions, used by `Fix` to detect that the fixpoint is reached. The solution is to define `property` as the number of instances generated so far along with the bound we have chosen in advance:

```
type variable = Atom : 'a ty -> variable
type property = { required : int; produced : int }
let equal p1 p2 = p1.produced = p2.produced
let is_maximal p = p.produced >= p.required
```

4. Expressing correctness properties

We mentioned earlier (§2) the `counter_example` function.

```
val counter_example: 'a pos -> ('a -> bool) -> 'a option
```

The function takes a description of some (positive) datatype `'a`, iterates on the generated instances of this type and checks that a predicate `'a -> bool` holds for all instances, or returns a counter-example otherwise. At a more abstract level, this means that we are checking a property of the form $\forall(x \in t), T(x)$ where $T(x)$ is simply a boolean expression. Multiple quantifiers can be simulated through the use of product types, such as in the typical formula of association maps:

$$\forall(m \in \mathtt{map}(K, V), k \in K, v \in V),\ \mathtt{find}(\mathtt{k}, \mathtt{add}(\mathtt{k}, \mathtt{v}, \mathtt{m})) = \mathtt{v}$$

which can be expressed as follows (where `*@` is the operator for creating product type descriptors):

```
let lookup_insert_prop (k, v, m) =
  lookup k (insert k v m) = v
let () = assert (None =
  let kvm_t = k_t *@ v_t *@ map_t in
  counter_example kvm_t lookup_insert_prop)
```

One then naturally wonders what a good language would be for describing the correctness properties we wish to check. In the example above, we naturally veered towards first-order logic, so as to express formulas with only prenex, universal quantification. The universal quantifiers are to be understood with a "test semantics", that is, they mean to quantify over all the random instances we generated. Can we do better? In particular, can we capture the full language of first-order logic, as a reasonable test *description language* for a practical framework?

It feels natural to use first-order logic as a specification language in the context of structured verification, such as with SMT solvers or a finite model finder [4]. However, supporting full first-order logic as a specification language for randomly-generated tests is hard for various reasons.

For instance, giving "test semantics" to an existentially-quantified formula such as $\exists(x \in t).T(x)$ is awkward. Intuitively, there is not much meaning to the formula. The number of generated instances is finite; that none satisfies T may not indicate a bug, but rather that the wrong elements have been tested for the property. Conversely, finding a counter-example to a universally-quantified formula *always* means that a bug has been found. Trying to distinguish absolute (positive or negative) results from probabilistic results opens a world of complexity that we chose not to explore.

Surprisingly enough, there does not seem to be a consensus in the literature about random testing for an expressive, well-defined subset of first-order logic. The simplest subset one directly thinks of is formulas of the form: $\forall x_1 \ldots x_n, P(x_1, \ldots, x_n) \Rightarrow T(x_1, \ldots, x_n)$ where $P(x_1, \ldots, x_n)$ (the *precondition*) and $T(x_1, \ldots, x_n)$ (the *test*) are both quantifier-free formulas.

The reason this implication is given a specific status is to make it possible to distinguish tests that succeeded because the *test* was effectively successful from tests that succeeded because the *precondition* was not met. The latter are "aborted" tests that bring no significant value, and should thus be disregarded. In ArtiCheck, we chose to restrict ourselves to this last form of formulas.

5. Examples

5.1 Red-black trees

The (abridged) interface exported by red-black trees is as follows. The module provides iteration facilities over the tree structure through the use of [8] *zippers*. Our data structures are persistent.

```
module type RBT = sig
  type 'a t
  val empty : 'a t
  val insert : 'a -> 'a t -> 'a t
  type direction = Left | Right
  (* type 'a zipper *)
  type 'a ptr (* = 'a t * 'a zipper *)
  val zip_open : 'a t -> 'a ptr
  val move : direction -> 'a ptr -> 'a ptr option end
```

This examples highlights several strengths of ArtiCheck.

First, two different types are involved: the type of trees and the type of zippers. While an aficionado of internal testing may use the `empty` and `insert` functions repeatedly to create new instances of `'a t`, it becomes harder to type-check calls to *either* `insert` or `zip_open`. Our framework, thanks to GADTs, generates instances of both types painlessly and automatically.

Second, we argue that a potential mistake is detected trivially by ArtiCheck, while it may turn out to be harder to detect using internal testing. If one removes the comments, the signature reveals that pointers into a tree are made up of a zipper along with a tree itself. It seems fairly natural that the developer would want to reveal the `zipper` type; it is, after all, a fundamental feature of the module. An undercaffeinated developer, when writing internal test functions, would probably perform sequences of calls to the various functions. What they would fail to do, however, is destructing pairs so as to produce a zipper associated with *the wrong tree*. This particularly wicked usage would probably be overlooked. ArtiCheck successfully destructs the pair and performs recombinations, thus finding the bug.

5.2 Binary Decision Diagrams

Binary Decision Diagrams (BDDs) represent trees for deciding logical formulas. The defining characteristic of BDDs is that they enforce *maximal sharing*: wherever two structurally equal subformulas appear, they are guaranteed to refer to the same object in memory. A consequence is that performing large numbers of function calls does not necessarily means using substantially more memory: it may very well be the case that significant sharing occurs.

We mentioned earlier that our strategy for external testing amounted, in essence, to representing series of well-typed function calls in the simply typed lambda calculus using in GADT. If we only did that and skipped section §3, externally-testing BDDs would be infeasible, as we would end up representing a huge number of function calls in memory.

Conversely, with the design we exposed earlier, we merely record new instances as they appear without holding the entire set of potential function calls in memory. This allows for an efficient, non-redundant generation of test cases (instances).

5.3 AVL trees

AVL trees are a classic of programming interviews; many a graduate student has been scared by the mere mention of them. It turns out that tenured professors *should* be scared too: the OCaml implementation of sets, written using AVL trees by a respectable researcher, contained a bug that went unnoticed for more than ten years. The bug was discovered when another enthusiastic researcher set out to formalize the said library in Coq. The bug was fixed, and all was well. Out of curiosity, we decided to run ArtiCheck on the faulty version of the library. After registering only four functions with ArtiCheck, the bug was correctly identified by our library, with arguably less pain than the full Coq formalization required.

6. Related and Future Work

Genericity of value generation The idea of generating random sequences of operations instead of random internal values is not novel; for example, QuickCheck was used as is to test imperative programs [6], by generating random values of an AST of operations, paired to a monadic interpreter of those syntactic descriptions. However, those examples in the literature only involve operations for a single return type, corresponding to the return type of the AST evaluation function. To integrate operations of distinct return types in the same interface description, one needs GADTs or some other form of type-level reasoning.

When multiple value types are involved, we found it helpful to think of well-typed value generation as term/proof search. Our well-typed rule to generate random values at type τ from a function at type $\sigma \to \tau$ and random values at type σ could be expressed, in term of QuickCheck `Arbitrary` instances, as a deduction rule of the form

```
instance Arbitrary b, Arbitrary (a -> b) => Arbitrary b
```

However, Haskell's type-class mechanism would not allow this instance deduction rule, as it does not respect its restrictions for principled, coherent instance elaboration. Type classes are a helpful and convenient host-language mechanism, but they are designed for code inference rather than arbitrary proof search. Our library-level implementation of well-typed proof search using GADTs gives us more freedom, and is the central idea of ArtiCheck.

It is of course possible to see chaining of function/method calls as a metaprogramming problem, and generate a reified description of those calls, to interpret through an external script or reflection/JIT capability, as done in the testing tool Randoop [10]. Doing the generation as a richly-typed host language library gives us stronger type safety guarantees: even if our value generator is buggy, it will never compose operations in a type-incorrect way.

Testing of higher-order or polymorphic interfaces The type description language we use captures a first-order subset of the simply-typed lambda-calculus. A natural question is whether it would be possible to support random function generation – embed negative types into positives. A simple way to generate a function of type `t -> u` is to just generate a `u` at each call; QuickCheck additionally uses the `t` argument to produce additional entropy. This is not completely satisfying as it does not use the argument at type `t` (which may not be otherwise reachable from the interface) to produce new values of type `u`. To have full test coverage for higher-order functional, one should locally add the argument to the set of known elements at type `t`, and re-generate values at type `u` in that extended environment.

It would also be interesting to support representation of polymorphic operations; we currently only describe monomorphic instantiations. Bernardy *et. al.* [3] have proposed a parametricity-based technique to derive specific monomorphic instances for type arguments, which also reduces the search space of values to be tested. Supporting this technique would be a great asset of a testing library, but it is definitely not obvious how their pen-and-paper derivation could be automatized, especially as a library function.

Bottom-up or top-down generation We have presented the `ArtiCheck` implementation as a bottom-up process: from a set of already-discovered values at the types of interest, we use the constructors of the interface to produce new values. In contrast, most random checking tools present generation in a top-down fashion: pick the head constructor of the data value, then generate its subcomponents recursively. One notable exception is SmallCheck [12], which performs exhaustive testing for values of bounded depth.

The distinction is however blurred by several factors. Fix implements demand-driven computation of fixpoints: if you request elements at type `u` and there is an operation `t -> u`, it will recursively populate values at type `t`, giving the actual operational behavior of the generator a top-down flavor. Relatedly, SmallCheck has a Lazy SmallCheck variant that uses laziness (demand-driven computation) to avoid fleshing out value parts that are not inspected by the property being tested.

Furthermore, the genericity of our high-level interface makes `Articheck` amenable to a change in the generation technique; we could implement direct top-down search without changing the signature description language, or most parts of the library interface.

Richer property languages We discussed in Section 4 the difficulty of isolating an expressive fragment of first-order logic as a property language that could be given a realizable testing semantics. As it performs exhaustive search (up to a bound), SmallCheck is able to give a non-surprising semantics to existential quantification. As we let user control for each interface datatype whether an exhaustive collection or a sampling collection should be used, we could support existential on exhaustively collected types only.

In related manner, Berghofer and Nipkow's implementation of QuickCheck for Isabelle [2] stands out by supporting full-fledged first-order logic for random checking. In the Isabelle proof assistant, it is common to define computations as inductive relations/predicates that can be given a (potentially non-deterministic) functional mode; instead of directly turning correctness formulas into testing programs, they translate formulas into inductive datatypes, which are then given a computational interpretation.

This is remarkable as it not only allows them to support a rich specification language, but also gives a principled explanation for the ad-hoc semantics of preconditions in testing frameworks (a failing precondition does not count as a passed test); instead of seeing a precondition `P[x]` as returning a boolean from a randomly-generated `x`, they choose a mode assignment that inverts it into a logic program generating the `xs` accepted by the precondition. This gives a logic-based justification to various heuristics used in other works to generate random values more likely to pass the precondition, either domain-specific [7] or SAT-based [1].

Conclusion

We have presented the design of ArtiCheck, a novel library that allows one to check the invariants of a module signature by simulating user interaction with the module. ArtiCheck behaves like a fake client: it calls functions, constructs and destructs products or sums, and for each element check that the invariants are verified. The key to performing this in a generic, abstract manner relies on GADTs, which abstract the different types that may be manipulated into a common representation.

We identified various performance problems that arise. The library handles them via a symbolic representation of types in combination with a little bit of mutable state to avoid handling large, intermediary results in memory.

The result is a self-contained library that wraps the core concepts of *external testing* and offers clients a cheap and efficient way to test their programs. The library, for instance, successfully detects infamous issues such as the AVL re-balancing issue in the standard library of OCaml, with a much lower cost than a complete machine-assisted verification of the module.

While the library exposes the essence of *external testing* and has already proven worthwhile, we believe there is potential for improvement and expansion into a fully-fledged testing library. The code, along with the entire history of the present paper, is available online at `https://github.com/braibant/articheck`.

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